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To cite this article: V A Tkachenko *et al* 2018 *J. Phys.: Conf. Ser.* **1092** 012154

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Transient time in nonlinear interaction of counter-propagating waves with the effect of pump absorption and phase mismatching

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Abstract. The parametric interaction of three waves, one of which travel in direction opposite to other, is associated with a number of unusual phenomena that find their applications. One of such phenomena is an extremely long transient process of amplitude changing of the output signal. In this paper, the impact of pump-wave absorption and the phase mismatch magnitude on the course of the process is investigated. It is shown that the absorption lead to significant changes in the process time, and the phase causes changes in dynamics but not in duration of the process.

1. Introduction

In recent years, in publications, much attention is paid to the possibilities of designing nanostructures with a unique electromagnetic response, which has prospects for use in various fields of both fundamental science and its various applications. Significant progress has been made in the field of nano engineering, and today it has become possible to create metamaterials with a negative refractive index (NIM) [1,2]. NIM, in turn, are comfort environments for the existence of backward waves, which are special waves whose phase vector direction and energy flux direction are opposite. These features open up profound entirely new possibilities of linear optics, unattainable with ordinary waves. The capabilities of NIM include: the creation of a sub-diffraction lens [3], sensitive sensors [4], optical cloaking [5], etc.

In the nonlinear optics, the backward waves in NIM are of interest as promising processes of parametric interaction of waves propagating counterwards each other - counterpropagating waves. The presence of a backward wave in this case, due to the inverse direction of the wave vector, makes it possible to reach phase-matching conditions in the medium [6]. The processes of counterpropagating waves interaction themselves are characterized by anomalous efficiency which can significantly exceed the conversion efficiency in the system of interaction with copropagating waves with the same values of nonlinearity and the medium length [7,8].

Another distinctive feature here is the presence of long transient processes[9]. As was shown in [10], the characteristic duration of a given process τ in the constant pump approximation $a_{30}KL < \pi/2$ also increases indefinitely with increasing gain parameters $\tau \sim 1/\cos(a_{30}KL)$ and can exceed, transient of copropagating case, by hundreds Δt . Such anomalous long-term processes should affect to the devices based on nonlinear NIMs. The influence of signal absorption and pump depletion on the process current was partially investigated earlier in [11, 12]. In particular, it was shown that



outside the constant pump approximation threshold $a_{30}KL > \pi/2$, the process time decreases with increasing gain parameters, and the process gets an abrupt form.

In this paper, some details of the medium absorption effect and phase mismatching on the transient process current are considered in more detail.

2. Model

Consider nonstationary problem three-wave interaction in one-dimensional quadratic nonlinear medium with length L . The magnitude of the electric field for each of the waves will be given by the expression $E_j(z, t) = A_j(z, t) \exp[i(\omega_j t - k_j z)]$ $j=1,2,3$, where $A_j(z, t)$ are the amplitudes; ω_j - wave frequencies, bounded by the relation $\omega_3 = \omega_1 + \omega_2$; k_j - wave vectors. The system of equations corresponding to the problem, and taking into account phase mismatch $\Delta k = k_3 - k_2 - k_1$, can be written as follows [11]:

$$\begin{cases} \partial a_1 / \partial z + (1/v_1) \partial a_1 / \partial t = +[iKa_3 a_2^* \exp(i\Delta kz) - (\alpha_1/2)a_1] \\ \partial a_2 / \partial z + (1/v_2) \partial a_2 / \partial t = -[iKa_3 a_1^* \exp(i\Delta kz) - (\alpha_2/2)a_2] \\ \partial a_3 / \partial z + (1/v_3) \partial a_3 / \partial t = -[iKa_1 a_2 \exp(-i\Delta kz) - (\alpha_2/2)a_3] \end{cases} \quad (1)$$

Here $a_j = (\omega_j)^{-1/2} (\epsilon_j)^{1/4} (\mu_j)^{-1/4} A_j$ is the normalized field amplitude; v_j - group velocity; $K = \chi^{(2)} 8\pi c^{-1} (\mu_1 \mu_2 \mu_3)^{1/4} (\epsilon_1 \epsilon_2 \epsilon_3)^{-1/4} (\omega_1 \omega_2 \omega_3)^{1/2}$ - coupling factor; $\chi^{(2)}$ - second-order nonlinear susceptibility of a medium; α_j is the absorption index of the medium at the frequency ω_j .

The wave with frequency ω_1 propagates in the negative direction of the axis, and the pump as well as the signal wave with frequencies ω_3 and ω_2 respectively propagate in the positive direction - this is determined by the signs in the equations.

To determine the temporal features of the transient process under absorption and phase mismatch conditions, the problem of the counterpropagating parametric amplifier was considered by equations (1) in the following configuration: The pump is continuously fed through the medium $a_3(z=0) = a_{30}$; the semi-infinite signal pulse in the form of a step $a_2(z=0) = a_{20}/2(1 - \tanh(-t/t_f))$ enters in the medium, where $a_{20} = 10^{-4} a_{30}$ - the input step amplitude, $t_f = 0.05L/v_3$ - the input step edge width, the counter wave is initially absent $a_1(z=L) = 0$. The problem was solved by numerical methods. The transient time τ was estimated as the time for which the output signal normalized amplitude was changed from a zero value to a $1 - e^{-1} \approx 0.63$ fraction of the steady state magnitude.

3. Absorption effect

Previous studies have shown that the problem has a characteristic threshold value of the gain parameters $a_{30}KL$ [10]. In the region below the threshold $a_{30}KL < Th$, the process time increases with the growth of the product of the parameters $a_{30}KL$, and it decreases in the region $a_{30}KL > Th$. In the absence of absorption $\alpha_1 = \alpha_2 = \alpha_3 = 0$, the threshold has a value $Th = \pi/2$ (Fig. 1 black circles). Also, in [11] it was shown that the stationary solution of system (1), obtained in [13], allows accurately predicting the position of the threshold with allowance for the amplified waves absorption. The stationary solution of the gain has the form:

$$\frac{a_{2L}}{a_{20}} = \frac{\exp((\alpha_1 - \alpha_2)L/2)}{\cos(RL) + \sin(RL)(\alpha_1 + \alpha_2)/4R} \quad R = \sqrt{K^2 - (\alpha_1 + \alpha_2)^2/16} \quad (2)$$

The singularity in expression (2) arises when the denominator turns to zero, which corresponds exactly to the threshold value, which has a singular character. So for the case

$\alpha_1 = \alpha_2 = \alpha_3 = 0$, we get $Th = \pi/2$, and for the case $\alpha_1 = \alpha_2 = 2.3L^{-1}$, $\alpha_3 = 0$ - $Th \approx 1.5111\pi/2$, which corresponds to the data of numerical simulation (Fig. 1. Gray circles). The threshold shift as a function of absorption is easily explained by the fact that in the presence of absorption, the same result requires a greater system gain efficiency.

Expression (2) does not allow to determine the precise threshold position in the pump absorption case. However, it can be assumed that an increase in pumping absorption will lead to a similar threshold shift further into the direction of large values $a_{30}KL$. In Fig. 2 calculated τ dependence for $\alpha_1 = \alpha_2 = 0$, $\alpha_3 = 2.3L^{-1}$ is shown by black crosses and for the case $\alpha_1 = \alpha_2 = \alpha_3 = 2.3L^{-1}$ by gray crosses. These dependences have thresholds $Th \approx 1.7\pi/2$ and $Th \approx 2.5\pi/2$ approximately, that confirms the proposed assumption.

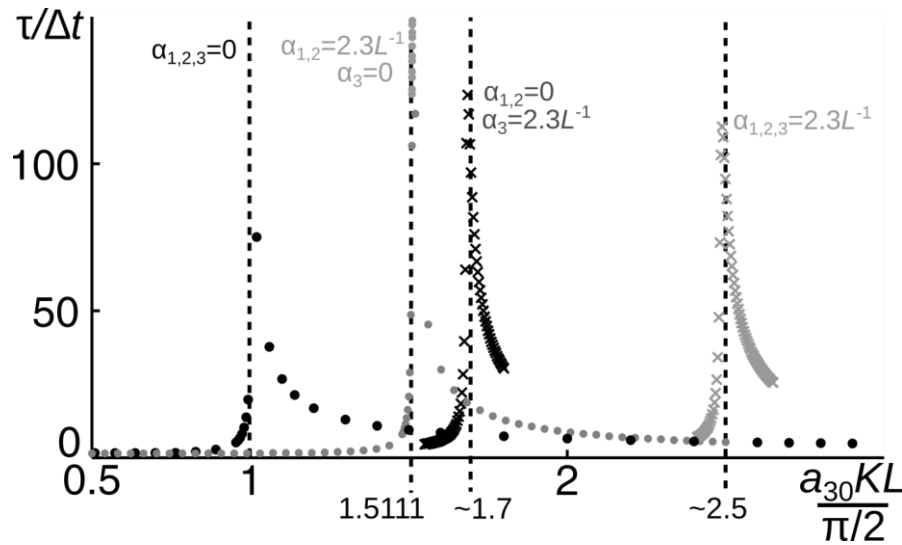


Figure 1. The dependence of the transient time τ on the gain parameters $a_{30}KL$:

$\alpha_1 = \alpha_2 = \alpha_3 = 0$ - black circles (•••); $\alpha_1 = \alpha_2 = 2.3L^{-1}$, $\alpha_3 = 0$ - gray circles (•••);
 $\alpha_1 = \alpha_2 = 0$, $\alpha_3 L = 2.3$ - black crosses (xxx); $\alpha_1 = \alpha_2 = \alpha_3 = 2.3L^{-1}$ - gray crosses (xxx).

Dashed lines show the corresponding threshold positions.

4. Phase mismatching effect

As a result of modeling, it was found that the change in the phase mismatch magnitude Δk does not affect the Th , but it affects the course of the transient process in the regions below $a_{30}KL < Th$ and above threshold $a_{30}KL > Th$.

In the region $a_{30}KL < Th$ the output signal decreases with increasing the steady state magnitude (see Fig. 3a). At the same time, its growth rate does not change, and damped oscillations of the normalized amplitude appear, and their period is linearly related to the mismatch value $T_{os} \sim 1/\Delta k$. The transient process time here increases with growth of Δk , according to the previously chosen formal evaluation criterion. However, such a criterion is well suited only for describing monotonic transient processes. For phase mismatch case, the time of the steady-state mode establishing actually increases due to oscillations. As can be seen from the graph, despite the same amplitude growth rate, the stationary mode is reached approximately at the same time for different values Δk .

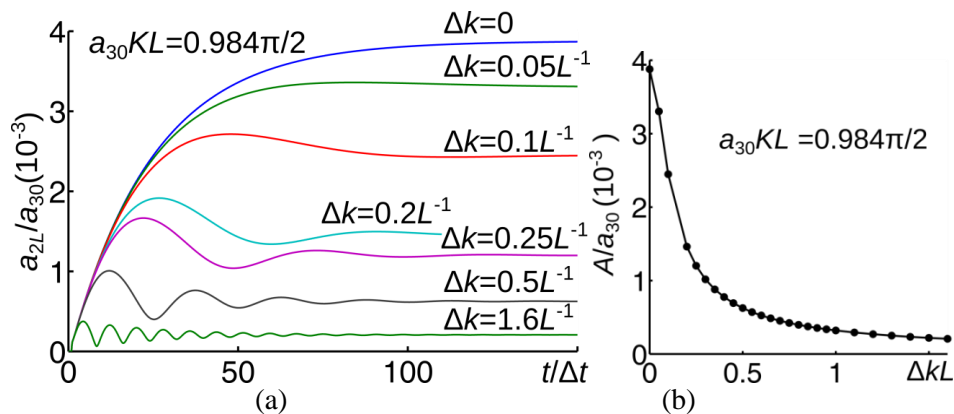


Figure 2. (a) Temporal behavior of the output signal normalized amplitude a_{2L}/a_{30} for different values of the phase mismatch magnitude Δk . (b) Dependence of the output signal normalized amplitude in the steady state mode A/a_{30} on the phase mismatch magnitude.

5. Conclusion

Thus, the absorption of the pump as well as the absorption of other waves leads to a shift in the threshold to a region of large values of $a_{30}KL$. This behavior is explained by the fact that achieving the same result requires a greater efficiency of amplification from the system.

However, the change in the phase mismatch magnitude Δk does not affect the position process threshold position. Moreover, in the region below the threshold, the process time does not decrease with growth of Δk due to the oscillations that arise, but this leads to a decrease in the signal level.

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Acknowledgments

The reported study was supported from Ministry of Education and Science of the Russian Federation (Grant № 3.6341.2017/6.7) , from RFBR according to the research (project № 18-32-00864) and by the U.S. Army Research Office (Grant Number W911NF-14-1-0619).